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Feasibility study of model-independent approach to ϕ_3 measurement using Dalitz plot analysis

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Abstract. In this paper, we present results of a feasibility study of a model-independent way to measure the angle ϕ_3 of the unitarity triangle. The method involves $B^\pm \to DK^\pm$ decays where the neutral D decays to the $K^0_S\pi^+\pi^-$ final state, together with the sample of decays of CP-tagged D mesons (produced, e.g. in $\psi(3770) \to D\bar{D}$ process) to the same final state. We consider different approaches to the extraction of ϕ_3 and obtain the expected statistical accuracy of the ϕ_3 measurement as a function of B and D_{CP} statistics.

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1 Introduction

Determinations of the Cabbibo–Kobayashi–Maskawa (CKM) [1] matrix elements provide important checks on the consistency of the Standard Model and ways to search for new physics. One of the ways to check the consistency of the CKM model is measurement of all angles of the unitarity triangle in decays of B mesons. One angle, ϕ_1, has been measured with high precision at modern B factories by the experiments BaBar [2] and Belle [3]. The measurement of the angle ϕ_2 is more difficult due to theoretical uncertainties in calculation of the penguin diagram contribution. Precise determination of the third angle, ϕ_3, is possible, e.g., in the decays $B^\pm \to DK^\pm$. Although it requires a lot more data than for the other angles, it is theoretically clean due to the absence of loop contributions. Finding the way to use the available experimental data most efficiently is therefore essential for the ϕ_3 measurement.

Various approaches have been proposed to measure ϕ_3 in $B^\pm \to DK^\pm$ decays [4–7] by utilizing the interference between $\bar{b} \to \bar{c}u\bar{s}$ and $\bar{b} \to \bar{u}c\bar{s}$ transitions. The ϕ_3 can be measured by studying the interference of $B^\pm \to D^0K^\pm$ and $B^\pm \to \overline{D}{}^0K^\pm$, which implies the use of neutral D final states common to D^0 and $\overline{D}{}^0$. This is provided by using CP eigenstates of D meson (GLW method [4]) or Cabibbo-favored and doubly Cabibbo-suppressed modes (ADS method [7]).

The Dalitz plot analysis of the three-body D^0 decay from $B^\pm \to DK^\pm$ processes [8–15] provides today the best measurement of the angle ϕ_3. It was proposed by Giri et al. [8] and independently by the Belle collaboration [9]. While Giri et al. mainly discussed a model-independent way of determining ϕ_3 using a binned Dalitz analysis of $\overline{D}{}^0 \to K^0_S\pi^+\pi^-$ decay from $B^\pm \to DK^\pm$, the Belle collaboration proposed an unbinned Dalitz plot fit of this decay using the $\overline{D}{}^0 \to K^0_S\pi^+\pi^-$ model based on a coherent sum of quasi-two-body amplitudes. The latter technique has been used so far by BaBar and Belle in their ϕ_3 measurement. The most recent BaBar result is $\phi_3 = 67° \pm 28°(\mathrm{stat}) \pm 13°(\mathrm{syst}) \pm 11°(\mathrm{model})$ using $211\,\mathrm{fb}^{-1}$ data sample [15], the result of the Belle measurement is $\phi_3 = 53°{}^{+15°}_{-18°}(\mathrm{stat}) \pm 3°(\mathrm{syst}) \pm 9°(\mathrm{model})$ with $357\,\mathrm{fb}^{-1}$ data sample [13]. The use of Cabibbo-favored mode $\overline{D}{}^0 \to K^0_S\pi^+\pi^-$ allows to increase the statistics compared to GLW or ADS methods. However, this approach suffers from the model uncertainty introduced by the unknown complex phase of D^0 decay. Currently, this uncertainty is estimated to be of order of 10°, which is lower than the statistical error. In the future, as the statistics of B decays collected at B-factories increases, this uncertainty will start to dominate the ϕ_3 measurement. While the estimations of the model uncertainty can also improve, they can only be based on arbitrary variations of the decay amplitude description within phenomenological models (e.g. changing the list of resonances or using K-matrix formalism). However these models are only approximations of the 3-body dynamics and thus any model uncertainty estimates cannot be treated as absolutely reliable.

It has been shown [8] that the D^0 model uncertainty can be eliminated using the sample of neutral D mesons in CP eigenstates. This sample can be obtained at charm facilities, like currently running CLEO-c [16], in the decays of $\psi(3770)$ into two neutral D mesons. In this paper, we report the results of a feasibility study of such an approach using toy Monte–Carlo (MC) simulation based on the realistic description of D decay amplitude. Our main goal is

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the determination of the amount of D_{CP} decays needed for the model-independent measurement.

The sensitivity to the angle ϕ_3 comes from the interference of two amplitudes producing opposite flavors of neutral D meson. Such a mixed state of neutral D will be called $\tilde{D}$. For example, in the case of B^+ decay, the mixed state is $\tilde{D}_+ = \overline{D}^0 + r_B e^{i\theta_+} D^0$. Here r_B is the ratio of the suppressed and favored amplitudes, the total phase between the $\overline{D}^0$ and D^0 is $\theta_+ = \phi_3 + \delta_B$, where δ_B is the strong phase difference between suppressed and favored B decays. Analogously, for the decay of B^-, one can write $\tilde{D}_- = D^0 + r_B e^{i\theta_-}\overline{D}^0$ with $\theta_- = -\phi_3 + \delta_B$. The ratio r_B is expected to be of order of 0.1–0.2 [17]. The BaBar ($r_B = 0.12 \pm 0.08 \pm 0.03(\text{syst}) \pm 0.04(\text{model})$, [14]) and Belle ($r_B = 0.16 \pm 0.05 \pm 0.01(\text{syst}) \pm 0.05(\text{model})$, [13]) measurements of the r_B in Dalitz analysis of D^0 from $B^\pm \to DK^\pm$ decay confirm these estimates.

The Dalitz plot density of $\tilde{D}$ gives immediate information about r_B and $\theta_\pm$, once the amplitude of the $\overline{D}^0$ decay is known. The amplitude of the $\tilde{D}_\pm$ decay as a function of Dalitz plot variables $m_+^2 = m^2_{K_S^0\pi^+}$ and $m_-^2 = m^2_{K_S^0\pi^-}$ is

$$f_{B^\pm} = f_D(m_\pm^2, m_\mp^2) + r_B e^{i\theta_\pm} f_D(m_\mp^2, m_\pm^2)\,, \qquad (1)$$

where $f_D(m_+^2, m_-^2)$ is an amplitude of the $\overline{D}^0 \to K_S^0\pi^+\pi^-$ decay. Here, we neglect possible effects of charm mixing and CP violation in D decay. It has been shown [18], that the mixing effects, which are suppressed in the Standard Model, lead to a bias in ϕ_3 only at a second order in the small parameters $\Delta m_D/\Gamma_D$ and $\Delta\Gamma_D/\Gamma_D$ (where Δm_D and $\Delta\Gamma_D$ are the mass and decay width differences between the two D mass eigenstates, Γ_D is the average decay width of mass eigenstates), and therefore can be safely neglected.

The $\overline{D}^0 \to K_S^0\pi^+\pi^-$ decay model can be determined from a large sample of flavor-tagged $\overline{D}^0 \to K_S^0\pi^+\pi^-$ decays produced in continuum e^+e^- annihilation. Once that is known, a simultaneous fit of B^+ and B^- data allows to separate the contributions of r_B, ϕ_3 and δ_B. However, flavor-tagged $\overline{D}^0 \to K_S^0\pi^+\pi^-$ decays only provide information about the absolute value of the f_D amplitude. The phase term can only be obtained based on some model assumptions (such as Breit-Wigner amplitude dependence of the underlying resonances) which results in the model uncertainty of the ϕ_3 measurement.

In the original publication by Giri et al. [8], an approach based on the binned Dalitz plot analysis was proposed which allows to extract ϕ_3 without model uncertainties. The idea of the approach is based on the fact that, to measure ϕ_3, complete knowledge of the phase term of f_D amplitude is not necessary. In the $\overline{D}^0 \to K_S^0\pi^+\pi^-$ decay amplitude, we emphasize the absolute value $|f|$ and phase δ_D:

$$f_D(m_+^2, m_-^2) = |f_D(m_+^2, m_-^2)| e^{i\delta_D(m_+^2, m_-^2)} \qquad (2)$$

Since the phase of f_B is arbitrary, it can be expressed as

$$f_{B^\pm} = |f_D(m_\pm^2, m_\mp^2)| + r_B e^{\pm i\phi_3 + i\delta_B} |f_D(m_\mp^2, m_\pm^2)| \\ \times \exp(i\delta_D(m_\pm^2, m_\mp^2) - i\delta_D(m_\mp^2, m_\pm^2))\,, \qquad (3)$$

i.e. only the difference $\delta_D(m_+^2, m_-^2) - \delta_D(m_-^2, m_+^2)$ of phase terms between symmetric points of the phase space is relevant.

This difference can be obtained either by studying the decays of neutral D in CP eigenstate or treating them as free parameters in the extraction of the ϕ_3 from the B decays (see Sect. 5). Decays of D mesons in CP eigenstate to $K_S^0\pi^+\pi^-$ can be obtained at e^+e^- machine in the process, e.g. $e^+e^- \to \psi(3770) \to D\bar{D}$, where the other (tag-side) D meson is reconstructed in CP eigenstate, such as K^+K^- or $K_S^0\omega$. The binned Dalitz plot analysis allows to determine ϕ_3 without the need to parametrize the dynamics of the $\overline{D}^0 \to K_S^0\pi^+\pi^-$ decay (e.g. by introducing resonances) and thus allows for a truly model independent measurement.

Different approaches to the extraction of δ_D phase and different Dalitz plot binnings are tested in our feasibility study and compared to the model-dependend approach performed in an unbinned fashion. We consider only the approaches described in [8]. However, another possibility exists to access the phase information, which involves the process $e^+e^- \to \psi(3770) \to D\bar{D}$ with both neutral D mesons decaying to $K_S^0\pi^+\pi^-$ final state [19]. This approach requires to study the correlations of the two Dalitz plots.

In the next section, we follow the technique proposed in [8], to introduce the notation which will be used in the rest of the paper.

2 Basic idea of the model-independent technique

The amplitude of $\overline{D}^0 \to K_S^0\pi^+\pi^-$ decay from CP eigenstate of neutral D is

$$f_{CP\pm}(m_+^2, m_-^2) = \frac{1}{\sqrt{2}}[f_D(m_+^2, m_-^2) \pm f_D(m_-^2, m_+^2)]\,, \qquad (4)$$

for CP-even and CP-odd states of neutral D, respectively.

Consider the $\overline{D}^0 \to K_S^0\pi^+\pi^-$ Dalitz plot is divided into $2\mathcal{N}$ bins symmetrically over exchange $m_+^2 \leftrightarrow m_-^2$. The bins will be denoted by the index i ranging from $-\mathcal{N}$ to $\mathcal{N}$ (excluding 0); the exchange $m_+^2 \leftrightarrow m_-^2$ corresponds to the exchange $i \leftrightarrow -i$. The number of events in the i-th bin of the Dalitz plot, given by the region $\mathcal{D}_i$, is

$$K_i = A_D \int_{\mathcal{D}_i} |f_D(m_+^2, m_-^2)|^2 \, dm_-^2 \, dm_+^2 \equiv A_D F_i\,, \qquad (5)$$

where A_D is the normalization that depends on the luminosity integral and definition of amplitude f_D.

Similarly, as follows from (1), for $\overline{D}^0 \to K_S^0\pi^+\pi^-$ decays from $B^\pm \to DK^\pm$ mode the number of events in i-th bin is

$$N_i^\pm = A_B \int_{\mathcal{D}_i} |f_{B^\pm}(m_+^2, m_-^2)|^2 \, dm_-^2 \, dm_+^2 \\ = A_B \left[\int_{\mathcal{D}_i} |f_D(m_\pm^2, m_\mp^2)|^2 \, dm_-^2 \, dm_+^2 \right.$$

$$
\begin{aligned}
&+ r_B^2 \int_{\mathcal{D}_i} |f_D(m_\mp^2, m_\pm^2)|^2 \, \mathrm{d}m_-^2 \, \mathrm{d}m_+^2 \\
&+ 2r_B \mathrm{Re} \left(e^{\mathrm{i}\theta_\pm} \int_{\mathcal{D}_i} f_D(m_+^2, m_-^2) \right. \\
&\left. \left. \times f_D^*(m_-^2, m_+^2) \, \mathrm{d}m_-^2 \, \mathrm{d}m_+^2 \right) \right] \\
&\equiv A_B \left[F_{\pm i} + 2r_B \mathrm{Re}\,(e^{\mathrm{i}\theta_\pm} G_i) + r_B^2 F_{\mp i} \right] \\
&= A_B \left[F_{\pm i} + 2r_B (\cos\theta_\pm \, \mathrm{Re}\, G_i - \sin\theta_\pm \, \mathrm{Im}\, G_i) \right. \\
&\left. + r_B^2 F_{\mp i} \right] . \qquad (6)
\end{aligned}
$$

Thus in order to extract information about $\theta_\pm$, one needs the values of $F_{\pm i}$, which are obtained from the decays of flavor tagged D mesons, and also the values of $\mathrm{Re}\, G_i$ and $\mathrm{Im}\, G_i$.

The number of events in i-th bin for CP eigenstate of neutral D is

$$
\begin{aligned}
M_i^\pm &= A_{CP\pm} \int_{\mathcal{D}_i} |f_{CP\pm}(m_+^2, m_-^2)|^2 \, \mathrm{d}m_-^2 \, \mathrm{d}m_+^2 \\
&= 2A_{CP\pm}(F_i \pm 2 \, \mathrm{Re}\, G_i + F_{-i}) . \qquad (7)
\end{aligned}
$$

Substituting the integrals F_i from (5) and introducing the parameters c_i and s_i which depend on the phase term $\phi(m_+^2, m_-^2)$, we finally obtain the following system of equations which relates the number of events in bins of flavor D, D_{CP} and D from B decays:

$$
\begin{aligned}
N_i^\pm &= h_B [K_{\pm i} + r_B^2 K_{\mp i} + 2r_B \sqrt{K_i K_{-i}} \\
&\quad \times (c_i \cos(\pm\phi_3 + \delta_B) - s_i \sin(\pm\phi_3 + \delta_B))] , \qquad (8) \\
M_i^\pm &= h_{CP\pm} [K_i + K_{-i} \pm 2\sqrt{K_i K_{-i}} c_i] , \qquad (9)
\end{aligned}
$$

where h_B and $h_{CP\pm}$ are the normalization factors which take into account different branching ratios, luminosity integrals and detection efficiencies for B, D^0 and $D_{CP\pm}$ decays,

$$
\begin{aligned}
c_i = \frac{1}{\sqrt{F_i F_{-i}}} &\int_{\mathcal{D}_i} |f_D(m_+^2, m_-^2)||f_D(m_-^2, m_+^2)| \\
&\times \cos(\delta_D(m_+^2, m_-^2) - \delta_D(m_-^2, m_+^2)) \, \mathrm{d}m_-^2 \, \mathrm{d}m_+^2 , \qquad (10)
\end{aligned}
$$

and

$$
\begin{aligned}
s_i = \frac{1}{\sqrt{F_i F_{-i}}} &\int_{\mathcal{D}_i} |f_D(m_+^2, m_-^2)||f_D(m_-^2, m_+^2)| \\
&\times \sin(\delta_D(m_+^2, m_-^2) - \delta_D(m_-^2, m_+^2)) \, \mathrm{d}m_-^2 \, \mathrm{d}m_+^2 . \qquad (11)
\end{aligned}
$$

Note that $c_i = c_{-i}$ and $s_i = -s_{-i}$. The meaning of these parameters is sine and cosine of the phase difference, averaged over the bin area with the weight proportional to $|f_D(m_+^2, m_-^2)||f_D(m_-^2, m_+^2)|$. It is clear that $s_i^2 + c_i^2 \le 1$.

The parameters c_i and s_i are in general unknown, they can be determined either from D_{CP} decays, or by leaving them as free parameters in the fit to D Dalitz plot from $B^\pm \to DK^\pm$ decay. Depending on the way these parameters are obtained, several approaches of ϕ_3 extraction are possible. Below we discuss these approaches, together with the toy MC simulation results for each of them.

In our toy MC studies, we assume that the statistics of flavor-tagged D mesons is practically infinite, therefore, the absolute value of the f_D amplitude is known to high precision. This is provided by the large cross-section of $D^{*\pm}$ meson production in $e^+e^- \to c\bar{c}$ process. We also assume that the number of D_{CP} decays is much larger than the number of B decays, due to the fact that the cross-section of $e^+e^- \to \psi(3770) \to D^0 D^0_{CP}$ production is approximately two orders of magnitude larger than that of $e^+e^- \to \Upsilon(4S) \to BB_{DK}$ process.

3 Approach with tagged D_{CP} and constraint on s_i

It is clear that for the most precise measurement of ϕ_3 given the limited statistics of B decays, all the available information has to be used. In particular, the unknown quantities c_i and s_i have to be extracted from the higher-statistics D_{CP} data.

The coefficient c_i can be obtained directly from the D^0 and D_{CP} decays. For example, if the numbers of D_{CP+} and D_{CP-} decays are proportional to the branching fractions of $\psi(3770) \to DD_{CP+}$ and $\psi(3770) \to DD_{CP-}$, respectively, *i.e.* if $h_{CP+} = h_{CP-}$,

$$
c_i = \frac{1}{2} \frac{M_i^+ - M_i^-}{M_i^+ + M_i^-} \frac{K_i + K_{-i}}{\sqrt{K_i K_{-i}}} \qquad (12)
$$

(see (9)). However, c_i can be extracted even in D decays of only one CP parity.

If the binning is fine enough so that both $f(m_+^2, m_-^2)$ and $f(m_-^2, m_+^2)$ are approximately constant over $(m_+^2, m_-^2) \in \mathcal{D}_i$, the expressions for c_i (10) and s_i (11) reduce to $c_i = \cos(\delta_D(m_+^2, m_-^2) - \delta_D(m_-^2, m_+^2))$, $s_i = \sin(\delta_D(m_+^2, m_-^2) - \delta_D(m_-^2, m_+^2))$. In such a case, s_i can be directly obtained as $s_i = \pm\sqrt{1 - c_i^2}$. The sign of s_i can be chosen either based on the continuity requirement (it should vary smoothly for adjacent bins) or under the assumption that the D^0 decay proceeds via two-body amplitudes. For simplicity, in our study we take the "correct" s_i sign (calculated from the input D^0 model).

Once the coefficients c_i and s_i are determined, the parameters r_B, ϕ_3 and δ_B can be obtained from B decays by minimizing the negative logarithmic likelihood

$$
-2 \log \mathcal{L} = -2 \log \mathcal{L}_+ - 2 \log \mathcal{L}_- , \qquad (13)
$$

where

$$
\begin{aligned}
-2 \log \mathcal{L}_\pm = \sum_{i=-\mathcal{N}}^{\mathcal{N}} \frac{1}{N_i^\pm} &(N_i^\pm - h_B [K_{\pm i} + r_B^2 K_{\mp i} \\
&+ 2r_B \sqrt{K_i K_{-i}} (c_i \cos(\pm\phi_3 + \delta_B) \\
&- s_i \sin(\pm\phi_3 + \delta_B))])^2 \qquad (14)
\end{aligned}
$$

is the negative logarithmic likelihood for one B flavor (B^+ or B^-), $N_i^{\pm}$ is the number of events in i-th bin for corresponding B flavor. Here, Gaussian behavior of the number of events in each bin is assumed.

To avoid possible biases due to non-constant $f(m_+^2, m_-^2)$, the bin size should be relatively small. The size of the bin is determined by the width of the most narrow structure in the Dalitz plot; in our case, it is the ω meson with $\Gamma = 8.4\,\mathrm{MeV/c^2}$; therefore, typical scale of phase variation is $\Delta m^2 = 0.013\,\mathrm{GeV^2/c^4}$. However, since we assume that the number of B decays is much smaller than the number of D_{CP} decays, the number of B events per bin for such a fine binning can be small. This can affect the fit procedure since we expect Gaussian behavior of the number of events. Therefore, for our study we take relatively rough binning for the fit of B decays, and introduce a sub-binning of each of "B-bins" into several "D-bins". As follows from definitions (10), (11) and (5), the parameters c_i and s_i for "B-bins" can be calculated from finer "D-bins" as

$$c_i = \sum_{\alpha} c_{i,\alpha}\sqrt{K_{i,\alpha}K_{-i,-\alpha}}/\sqrt{K_i K_{-i}}\,,$$
$$s_i = \sum_{\alpha} s_{i,\alpha}\sqrt{K_{i,\alpha}K_{-i,-\alpha}}/\sqrt{K_i K_{-i}}\,, \qquad (15)$$

where the index α corresponds to the sub-binning of the bin $\mathcal{D}_i$.

A toy MC study has been performed to obtain the accuracy of ϕ_3 measurement as a function of B and D_{CP} decay statistics, and to investigate the influence of the finite bin size on the measurement result. The Dalitz plot binning used for our MC study is shown in Fig. 1. The phase space $\Delta m^2 = m_+^2 - m_-^2 \in (-3\,\mathrm{GeV^2/c^4}, 3\,\mathrm{GeV^2/c^4})$ and $m_{\pi\pi}^2 = M_D^2 - 2M_\pi^2 - M_K^2 - m_+^2 - m_-^2 \in (0.2\,\mathrm{GeV^2/c^4})$ was uniformly divided into rectangular bins, symmetrically over the exchange of the sign of the final state pions. The bin size should certainly affect the statistical precision of the measurement. Since the structure of the $\overline{D}^0 \to K_S^0\pi^+\pi^-$ decay matrix element is quite complex, the variations of the event density due to CP violation change rapidly across the phase space, and, as the bin size becomes larger, the average effect of CP violation on each bin is reduced. Three different binnings were studied; depending on the number of bins in half of the phase space they are referred to as 3×3 (the corresponding number of bins which enter the half of the kinematically allowed phase space is $\mathcal{N} = 8$), 5×5 ($\mathcal{N} = 19$) and 7×7 ($\mathcal{N} = 32$) binnings.

The parameters r_B and δ_B were taken to be 0.2 and 180°, respectively; ϕ_3 was scanned in a range from 0° to 180°. For the D^0 decay model, the recent Belle measurement was used [12]. The efficiency variations over the phase space and background contribution were not taken into account in the simulation. The statistical precision of the method certainly depends on the actual D^0 model, therefore, the real experimental precision can somewhat differ from our estimate. The error is also inversely proportional to the magnitude r_B of the opposite D flavor contribution. The estimates of this value vary in the range 0.1–0.2, the current measurements of r_B [13, 15] agree with these numbers, although the error is still large. If the actual value of r_B differs from our expectation, the results of our study should be scaled accordingly.

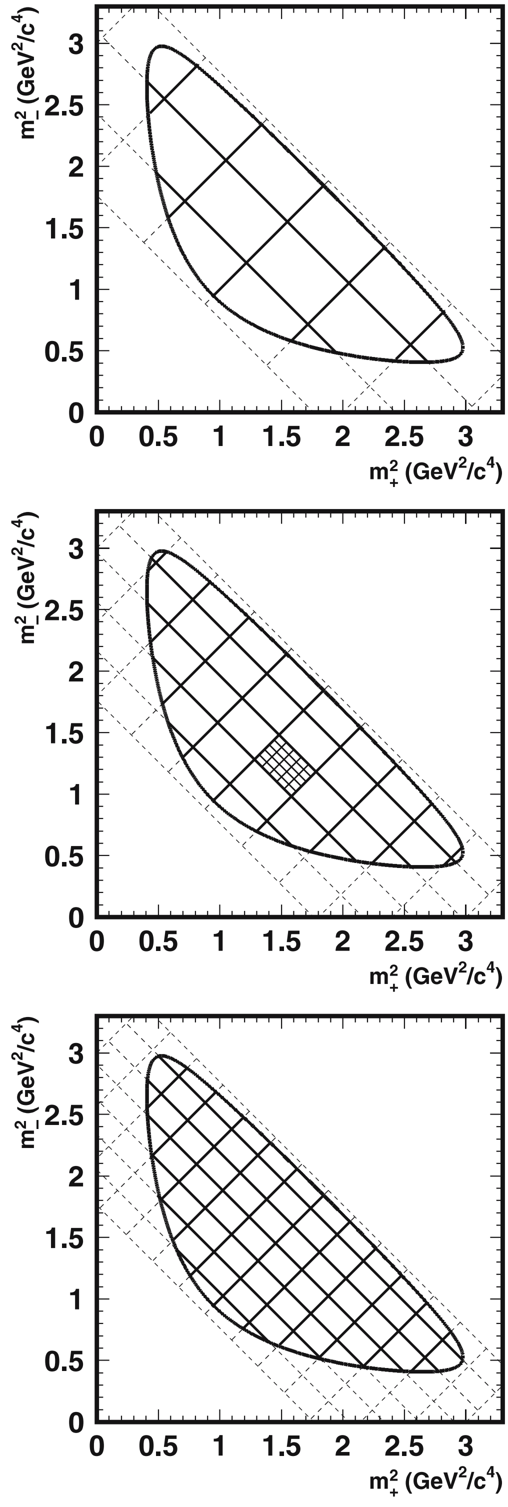

Fig. 1. Binning of the $\overline{D}^0 \to K_S^0\pi^+\pi^-$ Dalitz plot. 3×3 (*top*), 5×5 (*center*) and 7×7 binning (*bottom*). 5×5 sub-binning of one of the bins of the central plot is shown; the sub-binning is used for D_{CP} Dalitz plot

The statistical error of the ϕ_3 measurement as a function of ϕ_3 for the different binnings is shown in Fig. 2. For

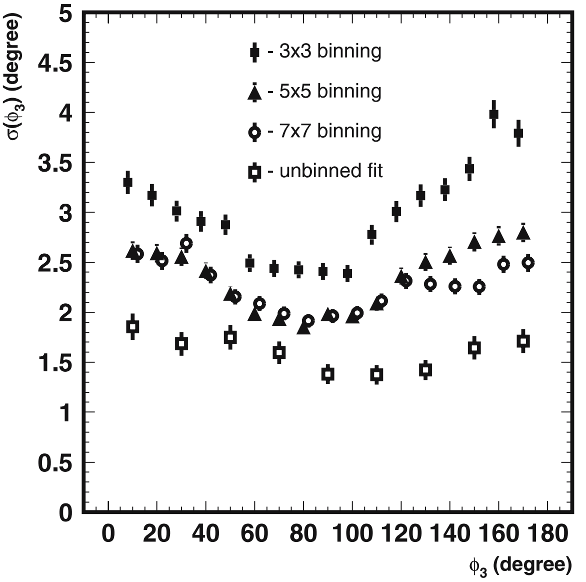

Fig. 2. ϕ_3 error as a function of ϕ_3, for different Dalitz plot binnings. Results of the toy MC study with 2×10^4 $B^\pm \to DK^\pm$ decays, 4×10^5 D_{CP} decays, $r_B = 0.2$, $\delta_B = 180°$

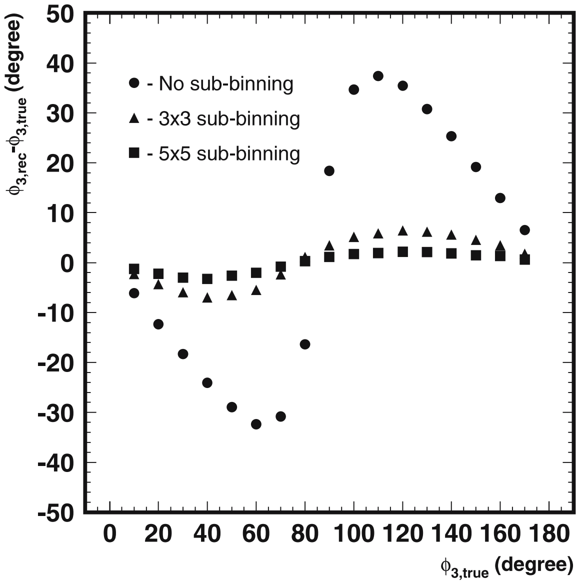

Fig. 3. ϕ_3 bias due to finite size of the bin as a function of ϕ_3, for the cases of no sub-binning (*circles*), 3×3 sub-binning (*triangles*), 5×5 sub-binning (*squares*)

each ϕ_3 value, 200 pseudo-experiments with the statistics of 10^4 $B^\pm \to DK^\pm$ decays were generated. The ϕ_3 error was obtained from the spread of the reconstructed values. To keep the contribution of finite D_{CP} statistics small, we used the large number of D_{CP} decays (2×10^5 decays of each CP-parity). The ϕ_3 accuracy depends only weakly on the values of ϕ_3 or δ_B since both c_i and s_i are obtained from the same D_{CP} statistics and have comparable accuracies. There is no significant difference in the statistical accuracy for 5×5 and 7×7 binnings, while in the case of 3×3 binning the accuracy is worse by 30%–40%. We have used the 5×5 binning for all further studies (excluding the study of the approach without D_{CP} decays, see Sect. 5). The precision of ϕ_3 extracted from a model-dependent unbinned Dalitz plot fit with the same input parameters is also shown for comparison. The unbinned fit is the optimal technique from the statistical point of view and represents the best limit of the statistical precision. The resolution in that case is approximately 30% better than for the case of binned approach. The optimal choice of binning can possibly improve the accuracy of the binned technique. We leave that question for the future studies.

The results of the toy MC study of ϕ_3 bias due to finite D-bin size are shown in Fig. 3. The angle ϕ_3 was scanned from 0 to 180°, other parameters were taken to be $r_B = 0.2$, $\delta_B = 180°$. Points show the ϕ_3 bias for no sub-binning, 3×3 sub-binning and 5×5 sub-binning (shown in Fig. 1 for one of the bins). For our further study, we have used 5×5 sub-binning, for which the bias does not exceed 2°. In a real experiment the bias can be corrected, e.g. based on the quasi two-body model. Since the correction is comparatively small, the uncertainty introduced by a model-based correction should be negligible.

The results of the study of ϕ_3 sensitivity dependence on the number of reconstructed $B^\pm \to DK^\pm$ and D_{CP} de-

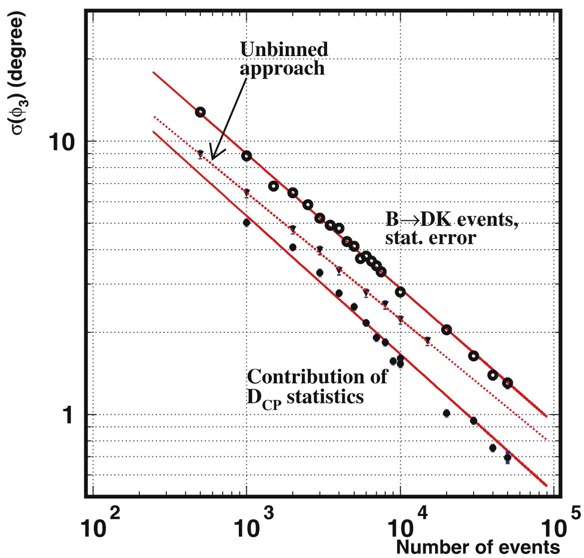

Fig. 4. Statistical error of ϕ_3 as a function of number of reconstructed $B^\pm \to DK^\pm$ decays (*open circles*) and D_{CP} decays (*dots*). Results of toy MC study with $r_B = 0.2$, $\phi_3 = 70°$, $\delta_B = 180°$. ϕ_3 error from model-dependent unbinned Dalitz plot fit with the same input parameters is also shown (*dotted line*)

cays are shown in Fig. 4. The graph shows the ϕ_3 error for $r_B = 0.2$, $\phi_3 = 70°$, $\delta_B = 180°$. For different values of r_B, the error scales inversely proportional to r_B. The weak dependence of the statistical error on the phases ϕ_3 and δ_B should also be noted, therefore, the actual ϕ_3 error can differ by 30%–50% depending on the true values of these phases. The dependences on both B and D_{CP} statistics show a square root scaling. For comparison, the statistical error of ϕ_3 extracted by model-dependent unbinned Dalitz

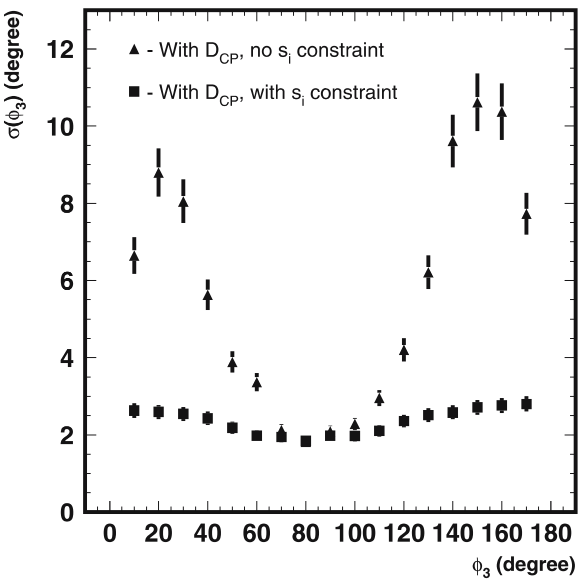

Fig. 5. ϕ_3 error as a function of ϕ_3 for relaxed s_i (*triangles*); and s_i constrained from c_i (*squares*). Results of the toy MC study with 2×10^4 $B^\pm \to DK^\pm$ decays, 4×10^5 D_{CP} decays, $r_B = 0.2$, $\delta_B = 180°$, 5×5 Dalitz plot binning

plot fit with the same input parameters is shown with the dotted line.

According to our estimations, the proposed super-B factory with its design integrated luminosity of $50\,\text{ab}^{-1}$, would allow a measurement of ϕ_3 with accuracy below 2°. To keep the contribution to the statistical error from the finite sample of D_{CP} decays below that level, one needs about 10^4 tagged D_{CP} decays, corresponding to approximately $10\,\text{fb}^{-1}$ of integrated luminosity at $\psi(3770)$ peak. In the following sections we consider possible alternative approaches of the binned analysis and compare their precision with the approach described above, which we have found to be the most optimal.

4 Approach without s_i constraint

Constraining s_i from c_i introduces a slight model dependence to the measurement result, since the bin size becomes dependent on the width of the Dalitz plot structures of D^0 decay. In principle, the system of equations is solvable even without this constraint. In this case, there are $8\mathcal{N}$ equations (8), (9) for $2\mathcal{N}+6$ unknowns (c_i, s_i, r_B, ϕ_3, δ_B, h_B, $h_{CP\pm}$). Coefficients c_i are still directly obtained from the D_{CP} decays, while s_i are mainly constrained by the B decays. Therefore, we expect the accuracy of the $\cos(\pm\phi_3 + \delta_B)$ determination to be higher than that for $\sin(\pm\phi_3 + \delta_B)$. In other words, the accuracy of ϕ_3 determination should strongly depend on the values of ϕ_3 and δ_B. The ϕ_3 error as a function of ϕ_3 for $r_B = 0.2$ and $\delta_B = 180°$ is shown in Fig. 5 (triangles). This plot was obtained by toy MC procedure with 10^4 $B^\pm \to DK^\pm$ decays of each flavor, and 2×10^5 D_{CP} decays of each CP-parity. For comparison, the ϕ_3 error is shown obtained with the approach where s_i is fixed from c_i. As follows from Fig. 5, the precision of the approach under study is comparable with the optimal strategy precision only in the narrow range of ϕ_3 values. The position of this region depends on the value of the strong phase δ_B, therefore, the precision of the ϕ_3 determination in this approach can not be guaranteed.

5 Approach without tagged D_{CP}

Even if the D_{CP} decays are not available, it is in principle possible to extract ϕ_3 in a model-independent way using only $B^\pm \to DK^\pm$ and flavor-tagged D decays. For $\mathcal{N}$ pairs of bins, there are $4\mathcal{N}$ equations (8) ($2\mathcal{N}$ for each flavor of B), and $2\mathcal{N}+4$ unknowns (c_i, s_i, r_B, ϕ_3, δ_B, h_B). Therefore, the system is solvable for $\mathcal{N} \geq 2$. Technically (if the number of equations is greater than the number of unknowns), it can be done by minimizing the negative logarithmic likelihood ((13), (14)) with c_i and s_i coefficients taken as free parameters.

Practically, however, this approach offers too low sensitivity for any reasonable B decay statistics. A toy MC study has been performed with 10^5 $B^\pm \to DK^\pm$ decays of each flavor. The parameters r_B and δ_B were taken to be 0.2 and 180°, respectively; ϕ_3 was scanned in a range from 30° to 140°. The results of the toy MC study are presented in Fig. 6 for 3×3 binning. Statistical precision with D_{CP} data and s_i constraint is also shown for comparison. It can be seen from the plots, that the accuracy of the ϕ_3 determination is quite poor compared to the method which makes use of D_{CP} decays. Reaching the accuracy of 2° would require about 5×10^6 reconstructed $B^\pm \to DK^\pm$

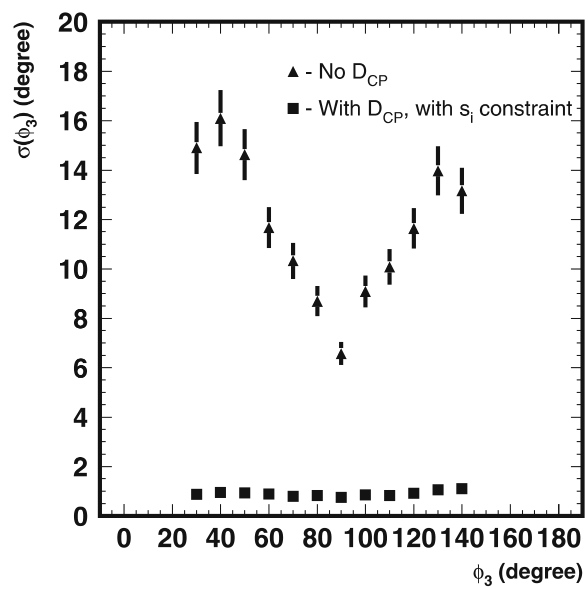

Fig. 6. ϕ_3 error as a function of ϕ_3 for the model-independent approach without D_{CP} decays (*triangles*) and with D_{CP} decays and s_i constraint (*squares*). Results of the toy MC study with 2×10^5 $B^\pm \to DK^\pm$ decays, $r_B = 0.2$, $\delta_B = 180°$, 3×3 Dalitz plot binning

decays, which corresponds to the currently unrealistic luminosity integral of 5000 ab^{-1} at B-factory.

Taking finer 5×5 binning could slightly improve the sensitivity, however, due to large number of free parameters, the fit converges poorly even with a number of B decays as large as 10^5. Variations of the initial parameters in the fit can lead to significant changes of the fit result. Binning finer than 5×5 has not been investigated due to an internal limitation on the maximum number of free parameters in the software used for minimization.

6 Conclusion

We have performed a toy Monte–Carlo study of a model-independent way to measure the angle ϕ_3 of the unitarity triangle, suggested by Giri et al. [8]. The method involves $B^\pm \to DK^\pm$ decays with neutral D decaying to $K^0_S\pi^+\pi^-$ final state, together with the decays of D mesons in CP eigenstate to the same final state. Different approaches to the extraction of ϕ_3 are considered. Although technically the model-independent measurement of ϕ_3 can be performed without decays of D in CP eigenstate, the statistics of $B^\pm \to DK^\pm$ decays required to obtain a reasonable ϕ_3 accuracy in this approach is unrealistically high.

We have shown that for the optimal strategy (which involves constraining s_i coefficients from c_i), the accuracy of the model-independent binned approach is only 30% worse compared to the unbinned technique. Possibly the choice of the optimal binning (e.g. using non-rectangular bins) can improve the accuracy of the model-independent measurement. The statistical error of ϕ_3 does not depend significantly on the ϕ_3 and strong phase values.

According to our estimations, the proposed super-B factory [20] with its design integrated luminosity of 50 ab^{-1}, would allow a measurement of ϕ_3 with statistical accuracy below 2°. To keep the contribution to the statistical error from the finite sample of D_{CP} decays below that level, around 10^4 tagged D_{CP} decays are needed, corresponding to approximately 10 fb^{-1} of integrated luminosity collected at $\psi(3770)$ peak. This task can be accomplished by CLEO-c experiment at CESR collider [16] and by upgraded BESIII/BEPCII complex at Beijing with design luminosity of $10^{33}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$ [21].

The systematic errors due to variations of the detection efficiency or background density over the Dalitz plot and other related factors were not considered in this study. They will certainly depend on the specific detector parameters, but the current experience with model-dependent Dalitz analyses at B factories allows us to believe that these contributions will also decrease with increasing statistics.

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